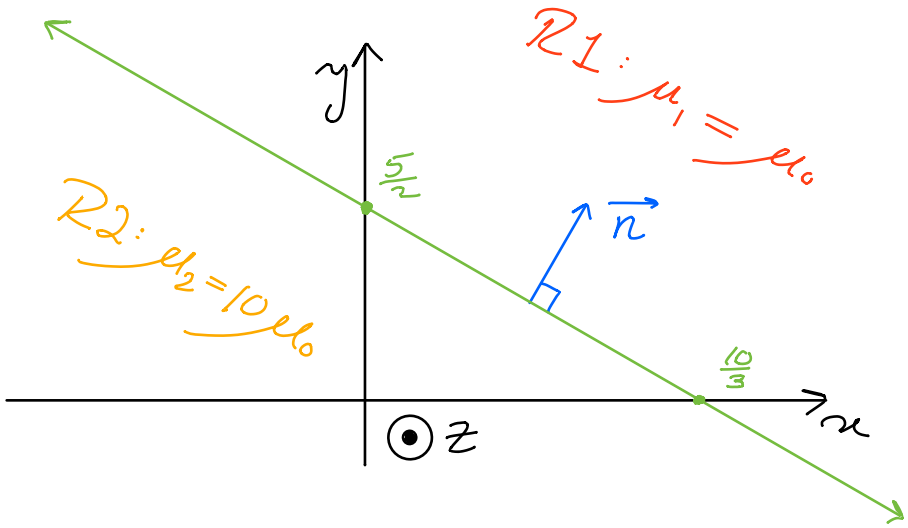
Princess Sumaya University for Technology
Communication Engineering Department
Electromagnetics I
Quiz 4

Region 1, described by $3x + 4y \geq 10$, is free space, whereas region 2, described by $3x + 4y \leq 10$, is a magnetic material for which $\mu = 10\mu_0$. Assuming that the boundary between the material and free space is current-free, find $\boldsymbol{B}_2$ if $\boldsymbol{B}_1 = 0.1\boldsymbol{a}_x + 0.4\boldsymbol{a}_y + 0.2\boldsymbol{a}_z$ Wb/m^2.



The boundary between the regions is current free: $K = 0$.

Recall: $H_{1t} - H_{2t} = \vec{K} \Rightarrow H_{1t} = H_{2t} \Rightarrow \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$

$\Rightarrow \frac{B_{1t}}{\mu_0} = \frac{B_{2t}}{10\mu_0} \Rightarrow B_{2t} = 10 B_{1t}$

Also $B_{2n} = B_{1n}$ But $B_{1n} = ?$, $B_{1t} = ?$

$\vec{B}_1 = 0.1\vec{a}_x + 0.4\vec{a}_y + 0.2\vec{a}_z = \vec{B}_{1t} + \vec{B}_{1n}$

we want to find $\vec{a}_n$: Normal unit vector to the boundary

The boundary equation is $3x + 4y = 10 \Rightarrow \vec{n} = 3\vec{a}_x + 4\vec{a}_y$

$\therefore \vec{a}_n = \frac{\vec{n}}{|\vec{n}|} = \frac{1}{5}(3\vec{a}_x + 4\vec{a}_y) = \frac{3}{5}\vec{a}_x + \frac{4}{5}\vec{a}_y$

Find $\vec{B}_{1n}$: Recall: $\vec{B}_{1n} = (\vec{B}_1 \cdot \vec{a}_n)\vec{a}_n$

$\therefore \vec{B}_{1n} = \left[(0.1\vec{a}_x + 0.4\vec{a}_y + 0.2\vec{a}_z) \cdot \left(\frac{3}{5}\vec{a}_x + \frac{4}{5}\vec{a}_y\right)\right]\vec{a}_n$

$= (0.06 + 0.32)\vec{a}_n = 0.38\left(\frac{3}{5}\vec{a}_x + \frac{4}{5}\vec{a}_y\right)$

$= (0.228\vec{a}_x + 0.304\vec{a}_y)\ Wb/m^2.$

Find $\vec{B}_{1t}$:

$$\vec{B}_1 = \vec{B}_{1n} + \vec{B}_{1t} \Rightarrow \vec{B}_{1t} = \vec{B}_1 - \vec{B}_{1n}$$

$$\Rightarrow \vec{B}_{1t} = (0.1\vec{a}_x + 0.4\vec{a}_y + 0.2\vec{a}_z) - (0.228\vec{a}_x + 0.304\vec{a}_y)$$

$$= (-0.128\vec{a}_x + 0.096\vec{a}_y + 0.2\vec{a}_z)\ Wb/m^2.$$

$$\therefore \vec{B}_{2n} = \vec{B}_{1n} = (0.228\vec{a}_x + 0.304\vec{a}_y)\ Wb/m^2$$

$$\therefore \vec{B}_{2t} = 10\vec{B}_{1t} = 10(-0.128\vec{a}_x + 0.096\vec{a}_y + 0.2\vec{a}_z)$$

$$= (-1.28\vec{a}_x + 0.96\vec{a}_y + 2\vec{a}_z)\ Wb/m^2$$

$$\therefore \vec{B}_2 = \vec{B}_{2t} + \vec{B}_{2n} = (-1.28\vec{a}_x + 0.96\vec{a}_y + 2\vec{a}_z) + (0.228\vec{a}_x + 0.304\vec{a}_y)$$

$$\therefore \vec{B}_2 = (-1.052\vec{a}_x + 1.264\vec{a}_y + 2\vec{a}_z)\ Wb/m^2.$$